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Pearson Edexcel
Level 3 GCE

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Mathematics

Advanced Subsidiary
Paper 3: Statistics and Mechanics

Sample Assessment Material for first teaching September 2017

Time: 2 hours

Paper Reference

9MA0/03**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

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Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

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SECTION A: STATISTICS

Answer ALL questions. Write your answers in the spaces provided.

1. *Kaff coffee* is sold in packets. A seller measures the masses of the contents of a random sample of 90 packets of *Kaff coffee* from her stock. The results are shown in the table below.

Mass w (g)	Midpoint y (g)	Frequency (f)
$240 \leq w < 245$	242.5	8
$245 \leq w < 248$	246.5	15
$248 \leq w < 252$	250	35
$252 \leq w < 255$	253.5	23
$255 \leq w < 260$	257.5	9

(You may use $\sum fy^2 = 5644171.75$)

A histogram is drawn and the class $245 \leq w < 248$ is represented by a rectangle of width 1.2 cm and height 10 cm.

- (a) Calculate the width and the height of the rectangle representing the class $255 \leq w < 260$ (3)
- (b) Use linear interpolation to estimate the median mass of the contents of a packet of *Kaff coffee* to 1 decimal place. (2)
- (c) Estimate the mean and the standard deviation of the mass of the contents of a packet of *Kaff coffee* to 1 decimal place. (3)

The seller claims that the mean mass of the contents of the packets is more than the stated mass.

Given that the stated mass of the contents of a packet of *Kaff coffee* is 250 g and the actual standard deviation of the contents of a packet of *Kaff coffee* is 4 g,

- (d) test, using a 5% level of significance, whether or not the seller's claim is justified. State your hypotheses clearly.

(You may assume that the mass of the contents of a packet is normally distributed.) (5)

- (e) Using your answers to parts (b) and (c), comment on the assumption that the mass of the contents of a packet is normally distributed.

(a) We are given that $245 \leq w < 248$ with freq. 15 has w 1.2 and h 10 cm
 \therefore area $1.2 \times 10 = 12 \text{ cm}^2 \rightarrow$ freq. 15 so 1 packet $= \frac{12}{15} = \frac{4}{5} \text{ cm}^2$ per packet! (M1)

The class is $248 - 245 = 3$ wide and 1.2 cm wide $\therefore \frac{1.2}{3} = 0.4 \text{ cm}$ width (per gram)

Hence for class $255 \leq w < 260$: 5 grams wide $\therefore 5 \times 0.4 = 2 \text{ cm}$ wide (B1)

9 packets $\therefore 9 \times \frac{4}{5} = \frac{36}{5} \text{ cm}^2$ area (A1)

$\therefore \frac{36}{5} \div 2 = 3.6 \text{ cm}$ height



Question 1 continued

(b) **Median** is Q_2 , the value in the **middle!**For our case since the **sample size** is 90, we need the 45th value!

Mass, w (g)	Midpoint	Frequency, f
$240 \leq w < 245$	242.5	8
$245 \leq w < 248$	246.5	15
$248 \leq w < 252$	250	35
$252 \leq w < 255$	253.5	23
$255 \leq w < 260$	257.5	9

cumulative frequencies

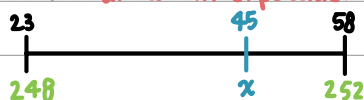
8

23

58

81

90

 \therefore the 45th value is in this class!Use **linear interpolation**

$$\frac{58-45}{58-23} = \frac{252-x}{252-248} \rightarrow \left(\frac{13}{35}\right) \times 4 = 252 - x \rightarrow x = 250.5 \text{ g } \text{A1} \quad \text{M1}$$

(c) **Formula** for mean:

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{(242.5 \times 8) + (246.5 \times 15) + (250 \times 35) + (253.5 \times 23) + (257.5 \times 9)}{90} = \frac{22535.5}{90} = 250.4 \text{ g } \text{B1}$$

Formula for SD:

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma_x = \sqrt{\frac{564471.75}{90} - \bar{x}^2} = \sqrt{15.64...} \text{ M1}$$

$$= 4.0 \text{ g } \text{A1}$$

(d) This part talks about "**mean**" \therefore we will use (sample mean) variable**Formula** for Sample mean:

$$X \sim N(\mu, \sigma^2) \rightarrow \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Apply the formula.enter $\frac{4}{\sqrt{90}} = \sigma$ into your calculator!

$$X \sim N(250, 4^2) \rightarrow \bar{X} \sim N\left(250, \frac{4^2}{90}\right) \text{ M1}$$

Hypotheses

$$P(\bar{X} > 250.4) = 0.171 > 0.05$$

 \therefore 250.4 does not fall in the **critical region**

$$H_0: \mu = 250 \text{ B1} \quad \text{A1} \quad \text{A1}$$

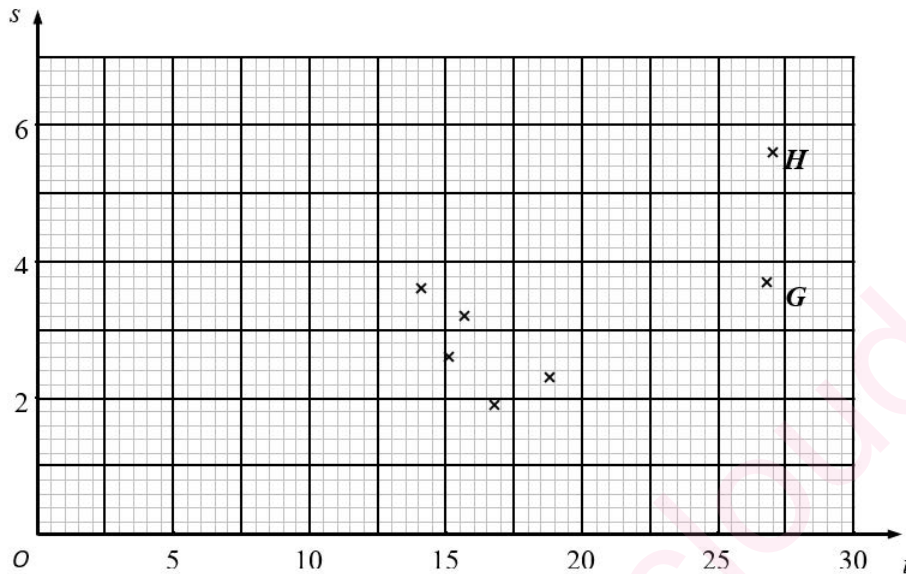
and there is insufficient evidence to reject H_0 .

$$H_1: \mu > 250$$

The claim is **unsupported** A1(e) It's **consistent** as the **mean** is close to the **median**, which is true for **normal distribution** B1

(Total for Question 1 is 14 marks)

2. A researcher believes that there is a linear relationship between daily mean temperature and daily total rainfall. The 7 places in the northern hemisphere from the large data set are used. The mean of the daily mean temperatures, t °C, and the mean of the daily total rainfall, s mm, for the month of July in 2015 are shown on the scatter diagram below.



- (a) With reference to the scatter diagram, explain why a linear regression model may not be suitable for the relationship between t and s . (1)

The researcher calculated the product moment correlation coefficient for the 7 places and obtained $r = 0.658$

- (b) Stating your hypotheses clearly, test at the 10% level of significance, whether or not the product moment correlation coefficient for the population is greater than zero. (3)
- (c) Using your knowledge of the large data set, suggest the names of the 2 places labelled G and H . (1)
- (d) Using your knowledge from the large data set, and with reference to the locations of the 2 places labelled G and H , give a reason why these places have the highest temperatures in July. (1)
- (e) Suggest how you could make better use of the large data set to investigate the relationship between daily mean temperature and daily total rainfall. (1)

(a) It's not suitable since the points don't lie on a straight line B1

(b) Hypotheses from tables, $p = 0.1$: $c.v. = 0.5509 < 0.658$ since the PMCC found is larger than the critical value, there's sufficient evidence to reject H_0 and $PMCC > 0$.

$H_0: \rho = 0$ M1

$H_1: \rho > 0$ B1 A1

Question 2 continued

(c) Beijing and Jacksonville B1

(d) Since they're the closest to the equator B1

(e) Only use data from one place. B1

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Question 2 continued

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Question 2 continued

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(Total for Question 2 is 7 marks)

3. For a particular type of bulb, $p=0.36$ 36% grow into plants with blue flowers and the remainder grow into plants with white flowers. Bulbs are sold in mixed bags of 40. Sample size, n

Russell selects a random sample of 5 bags of bulbs. $100-36=64\%$
 $5 \times 40 = 200$ bulbs

- (a) Find the probability that fewer than 2 of these bags will contain more bulbs that grow into plants with blue flowers than grow into plants with white flowers

(4)

Maggie takes a random sample of n bulbs.

Using a normal approximation, the probability that more than 244 of these n bulbs will grow into blue flowers is 0.0521 to 4 decimal places.

- (b) Find the value of n .

(6)

(a) $B \rightarrow$ # of bulbs that grow blue flowers

$$B \sim B(40, 0.36) \quad \text{M1}$$

$$p = P(B \geq 21) = 0.0240 \quad \text{A1}$$

we have 40 and want more than half to be blue.

$C \rightarrow$ # of bags with with more than 20 blue bulbs that are blue

$$C \sim B(5, p) \quad \text{M1}$$

$$P(C < 2) = P(C \leq 1)$$

$$= 0.9945 \rightarrow 0.995 \text{ to 3sf} \quad \text{A1}$$

(b) $T \rightarrow$ # of bulbs that grow blue flowers

$$T \sim B(n, 0.36)$$

$$\text{Mean formula: } np = 0.36n$$

$$\text{Variance formula: } np(1-p) = 0.36n(0.64) = 0.2304n$$

$$\text{Normal approx: } Y \sim N(0.36n, 0.2304n) \quad \text{B1}$$

$$\therefore P(T > 244) = 0.0521 \rightsquigarrow 1 - P(T \leq 244) = 0.0521, P(T \leq 244) = 0.9479 \quad \text{Binomial}$$

$$P(Y < 244.5) = 0.9479 \quad \text{normal}$$

Convert to Standard Normal, $Z \sim N(0, 1^2)$

$$P\left(Z < \frac{244.5 - 0.36n}{\sqrt{0.2304n}}\right) = 0.9479 \quad \text{M1}$$

$$\frac{244.5 - 0.36n}{\sqrt{0.2304n}} = \text{InvN}(0.9479) = 1.625 \quad \text{M1A1}$$

$$244.5 - 0.36n = 1.625\sqrt{0.2304n} \quad \text{solve for } n.$$

$$\text{M1 } 0 = 0.36n + 0.78\sqrt{n} - 244.5 \quad \text{hidden quadratic! } (x = \sqrt{n}, x^2 = n)$$

$$\text{Use Quadratic Formula: } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} : \sqrt{n} = \frac{-0.78 \pm \sqrt{(0.78)^2 - 4(0.36)(-244.5)}}{0.72} \rightarrow n = 625 \quad \text{A1}$$

Question 3 continued

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Lined writing area for the answer.

(Total for Question 3 is 10 marks)

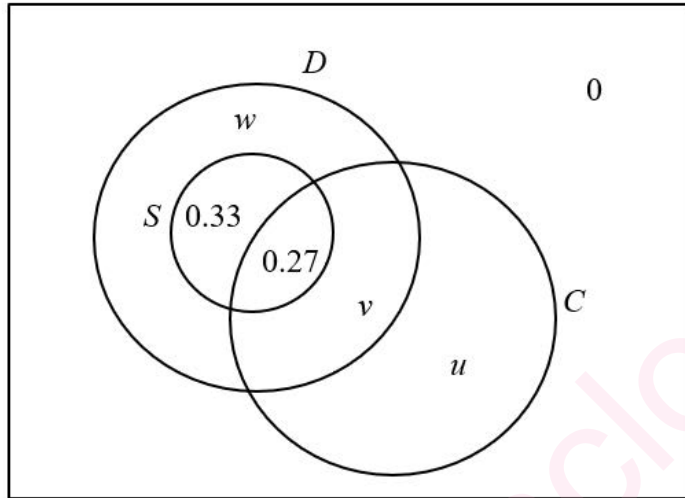
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4. The Venn diagram shows the probabilities of students' lunch boxes containing a drink, sandwiches and a chocolate bar.

- D is the event that a lunch box contains a drink,
- S is the event that a lunch box contains sandwiches,
- C is the event that a lunch box contains a chocolate bar,
- u, v and w are probabilities.



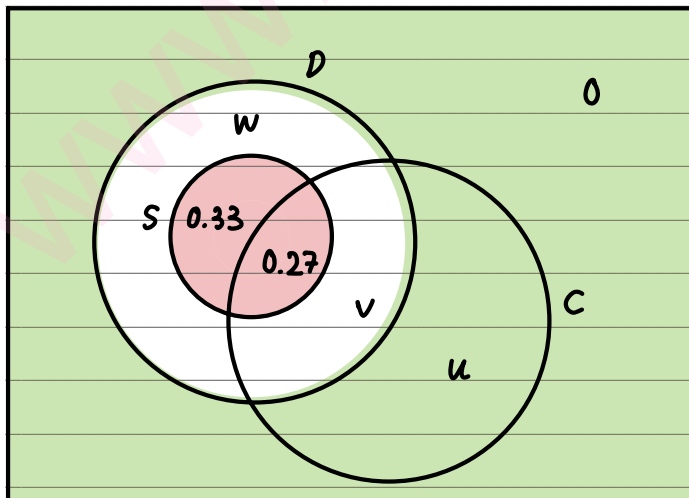
(a) Write down $P(S \cap D')$ (1)

One day, 80 students each bring in a lunch box.
Given that all 80 lunch boxes contain sandwiches and a drink,

(b) estimate how many of these 80 lunch boxes will contain a chocolate bar. (3)

Given that the events S and C are independent and that $P(D | C) = \frac{14}{15}$

(c) calculate the value of u , the value of v and the value of w . (7)



(a) $P(S \cap D')$
 $P(S)$ shaded red
 $P(D')$ shaded green
 \rightarrow We want what's shaded both green + red
 \rightarrow We see that the two do not intersect
 $\therefore P(S \cap D') = 0$ B1

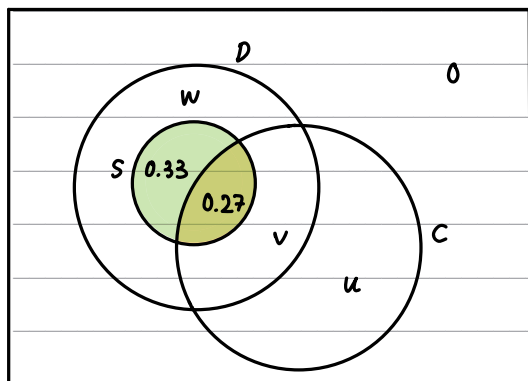
Question 4 continued

(b) "Given that..." \therefore we want $P(C|S \cap D)$

Formula for "Given":

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(C|S \cap D) = \frac{P(C \cap S \cap D)}{P(S \cap D)}$$



$$\therefore P(C \cap S \cap D) = 0.27$$

$$P(S \cap D) = 0.6$$

$$\therefore P(C|S \cap D) = \frac{0.27}{0.6} = 0.45 \quad \text{M1}$$

we have 80 lunchboxes:

$$0.45 \times 80 = 36 \quad \text{M1A1}$$

 $P(C \cap D \cap S)$ & $P(S \cap D)$

(c) Formula for independent events:

$$P(A) \times P(B) = P(A \cap B)$$

$$P(C) \times P(S) = P(C \cap S)$$

$$P(S) = 0.6$$

$$P(C) = 0.27 + v + u$$

$$P(S \cap C) = 0.27 \quad \text{M1}$$

Substitute

$$0.6(0.27 + v + u) = 0.27$$

$$u + v = 0.18 \quad \text{Eq.1}$$

Formula for "Given":

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(D|C) = \frac{P(D \cap C)}{P(C)}$$

$$P(D \cap C) = 0.27 + v$$

$$P(C) = 0.27 + v + u$$

Substitute:

$$\frac{14}{15} = \frac{0.27 + v}{0.27 + v + u} \rightarrow 14u - v = 0.27 \quad \text{Eq.2}$$

Solve simultaneously Eq1 + Eq2

$$u + v = 0.18$$

$$14u - v = 0.27 \quad +$$

$$15u = 0.45 \quad \text{ddM1}$$

$$u = 0.03, v = 0.15 \quad \text{A1}$$

 Σ probabilities = 1

$$0.6 + w + u + v = 1 \rightarrow 0.6 + w + 0.03 + 0.15 = 1$$

$$w = 0.22 \quad \text{A1}$$

(Total for Question 4 is 11 marks)

5. The lifetimes of batteries sold by company X are normally distributed, with mean 150 hours and standard deviation 25 hours.

A box contains 12 batteries from company X .

- (a) Find the expected number of these batteries that have a lifetime of more than 160 hours. (3)

The lifetimes of batteries sold by company Y are normally distributed, with mean 160 hours and 80% of these batteries have a lifetime of less than 180 hours.

- (b) Find the standard deviation of the lifetimes of batteries from company Y . (3)

Both companies sell their batteries for the same price.

- (c) State which company you would recommend. Give reasons for your answer. (2)

(a) $L_x \rightarrow$ lifetime of batteries from company X **define variable**

$$L_x \sim N(150, 25^2)$$

$$P(L_x > 160) = 0.34457 \rightarrow 0.345 \text{ to 3sf } \text{B1}$$

To get **expected** number of batteries

$$0.345 \times 12 = 4.13 \text{ batteries } \text{M1A1}$$

(b) $L_y \rightarrow$ lifetime of batteries from company Y **define variable**

$$L_y \sim N(160, \sigma^2)$$

$$P(L_y < 180) = 0.80 \text{ B1}$$

Convert to standard normal, $N(0, 1^2)$

$$P\left(z < \frac{180-160}{\sigma}\right) = \text{Inv}N(0.8) = 0.8416 \text{ M1}$$

$$2.0 = 0.8416\sigma$$

$$\sigma = 23.8 \text{ Standard deviation } \text{A1}$$

(c) The sd's are **similar** (23.8 and 25). **M1**

But for company Y the mean is higher \therefore **choose company Y** **A1**

Question 5 continued

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(Total for Question 5 is 8 marks)

TOTAL FOR SECTION A IS 50 MARKS

SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

[In this question position vectors are given relative to a fixed origin O .]

6. A particle, P , moves with constant acceleration $(\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-2}$.

At time $t = 0$ seconds, the particle is at the point A with position vector $(2\mathbf{i} + 5\mathbf{j}) \text{ m}$ and is moving with velocity $\mathbf{u} \text{ m s}^{-1}$.

At time $t = 3$ seconds, P is at the point B with position vector $(-2.5\mathbf{i} + 8\mathbf{j}) \text{ m}$.

Find \mathbf{u} .

(4)

Method 1 - SUVAT

$$r_0 = (2\mathbf{i} + 5\mathbf{j})\text{m}$$

$$r = (-2.5\mathbf{i} + 8\mathbf{j})\text{m}$$

$$\mathbf{u} = \mathbf{u}$$

$$x$$

$$\mathbf{a} = (\mathbf{i} - 2\mathbf{j})\text{ms}^{-2}$$

$$t = 3$$

Use formula:

$$r = r_0 + ut + \frac{1}{2}at^2 \quad \text{M1}$$

Substitute:

$$(-2.5\mathbf{i} + 8\mathbf{j}) = (2\mathbf{i} + 5\mathbf{j}) + 3\mathbf{u} + \frac{1}{2}(\mathbf{i} - 2\mathbf{j})(3)^2 \quad \text{B1}$$

$$(-2.5\mathbf{i} + 8\mathbf{j}) - (2\mathbf{i} + 5\mathbf{j}) = 3\mathbf{u} + (\frac{9}{2}\mathbf{i} - 9\mathbf{j})$$

$$(-4.5\mathbf{i} + 3\mathbf{j}) = 3\mathbf{u} + (4.5\mathbf{i} - 9\mathbf{j}) \quad \text{A1}$$

$$(-9\mathbf{i} + 12\mathbf{j}) = 3\mathbf{u}$$

$$\mathbf{u} = -3\mathbf{i} + 4\mathbf{j} \quad \text{A1}$$

Question 6 continued

Method 2 - IntegrationTo get **velocity** from **acceleration**, we need to **integrate**

$$v = \int a dt = \int (i - 2j) dt$$

$$= ti - 2tj + c$$

★ Simple Integration:

$$\int x^n dx = \frac{1}{n} x^{n+1} + c$$

To get **position vector** from **velocity**, **integrate again**

$$r = \int v dt = \int (t + c)i + (c_j - 2t)j dt$$

$$= \left(\frac{1}{2}t^2 + c_i t\right)i + (c_j t - t^2)j + k \quad M1$$

Now we **substitute** to get **c** and **k** B1 $t=0$, $r = (2i + 5j)m$: (to get **k**)

$$(2i + 5j) = \left(\frac{1}{2}t^2 + c_i t + k_i\right)i + (c_j t - t^2 + k_j)j$$

$$i. \quad 2 = \frac{1}{2}(0)^2 + c_i(0) + k_i \quad \therefore k_i = 2$$

$$j. \quad 5 = c_j(0) - (0)^2 + k_j \quad \therefore k_j = 5 \quad \therefore k = (2i + 5j)$$

 $t=3$, $r = (-2.5i + 8j)m$: (to get **c**)

$$(-2.5i + 8j) = \left(\frac{1}{2}t^2 + c_i t + k_i\right)i + (c_j t - t^2 + k_j)j$$

$$i. \quad -2.5 = \frac{1}{2}(3)^2 + c_i(3) + 2 \quad -9 = 3c_i \quad c_i = -3$$

$$j. \quad 8 = c_j(3) - 3^2 + 5 \quad 12 = 3c_j \quad c_j = 4 \quad \therefore c = (-3i + 4j)$$

A1

Complete **velocity** equation: $v = (t - 3)i + (4 - 2t)j$ Hence now we can substitute $t=0$ to get **u**:

$$u = (-3i + 4j) \quad A1$$

(this is not mentioned in the mark scheme but is still correct!)

(Total for Question 6 is 4 marks)

7. A particle, P , moves under the action of a single force in such a way that at time t seconds, where $t \geq 0$, its velocity \mathbf{v} m s⁻¹ is given by

$$\mathbf{v} = (t^2 - 3t) \mathbf{i} - 12t \mathbf{j}$$

The mass of P is 0.5 kg.

Find the time at which the magnitude of the force acting on P is 6.5 N.

(7)

We need to get acceleration to be able to use $\Sigma F = ma$.

So we differentiate M1

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = (2t - 3) \mathbf{i} - 12 \mathbf{j} \quad \text{a} \quad \text{A1}$$

Let force P be $(a\mathbf{i} + b\mathbf{j})$ N

We can substitute into $\Sigma F = ma$

$$(a\mathbf{i} + b\mathbf{j}) = 0.5(2t - 3)\mathbf{i} - 12\mathbf{j}$$

Now equate the separate components:

$$a = 0.5(2t - 3)$$

$$b = -6\mathbf{j}$$

To get the magnitude of a force use the Pythagoras' Theorem:

$$|a\mathbf{i} + b\mathbf{j}| = 6.5 = \sqrt{(0.5(2t - 3))^2 + (-6)^2} \quad \text{M1 A1}$$

$$6.5^2 = 0.25(4t^2 - 12t + 9) + 36$$

$$42.25 = t^2 - 3t + 2.25 + 36$$

$$0 = t^2 - 3t + 4 \quad \text{factorize} \quad \text{A1}$$

$$0 = (t - 4)(t + 1) \quad \text{M1}$$

$$t = 4 \quad \text{A1} \quad t = -1 \quad \text{reject as time can't be negative}$$

Question 7 continued

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(Total for Question 7 is 7 marks)

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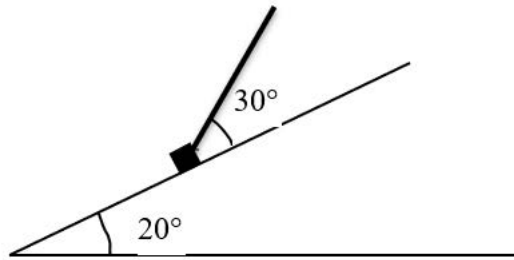


Figure 1

A small box of mass 3 kg moves on a rough plane which is inclined at an angle of 20° to the horizontal.

The box is pulled up a line of greatest slope of the plane using a rope which is attached to the box.

The rope makes an angle of 30° with the plane, as shown in Figure 1.

The rope lies in the vertical plane which contains a line of greatest slope of the plane.

The coefficient of friction between the box and the plane is 0.3.

The tension in the rope is 25 N.

The box is modelled as a particle, the rope is modelled as a light inextensible string and air resistance is ignored.

(a) Using the model, find the acceleration of the box.

(7)

(b) Suggest one improvement to the model that would make it more realistic.

(1)

The rope now breaks and the box slows down and comes to rest.

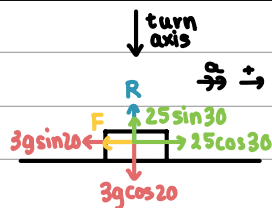
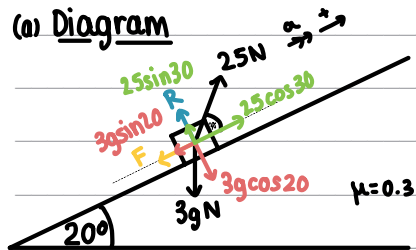
(c) Show that, after the box comes to rest, it immediately starts to move down the plane.

(3)

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Question 8 continued

(a) Diagram



To get A lets use $\Sigma F_x = ma$: M1

$$25\cos 30 - F - 3g\sin 20 = 3a \quad \text{A1}$$

Since it's moving, $F = \mu R$. B1

Get R using $\Sigma F_y = 0$ M1

$$R + 25\sin 30 = 3g\cos 20 \quad \text{A1}$$

$$R = 3g\cos 20 - 25\sin 30$$

$$F = 0.3(3g\cos 20 - 25\sin 30)$$

Substitute this. M1

$$25\cos 30 - 0.3(3g\cos 20 - 25\sin 30) - 3g\sin 20 = 3a$$

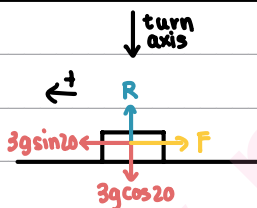
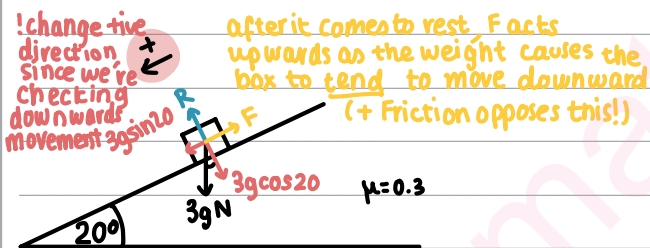
Solve for a: (enter into your calculator)

$$3a = 7.057$$

$$a = 2.35 \text{ m/s}^2 \quad \text{A1}$$

(b) Include air resistance B1

(c) Redraw Diagram without rope



Let's get $F_{\text{max}} = \mu R$

We need to get R using $\Sigma F_y = 0$

$$R = 3g\cos 20$$

Substitute:

$$F = 0.3 \times 3g\cos 20 \quad \text{B1}$$

$$F = 8.28 \text{ force to the right}$$

$$3g\sin 20 = 10.55 \text{ force to the left}$$

Hence as

$$F_{\text{max}} < W_x \quad \text{M1}$$

the box slides down the plane A1

(Total for Question 8 is 11 marks)

9.

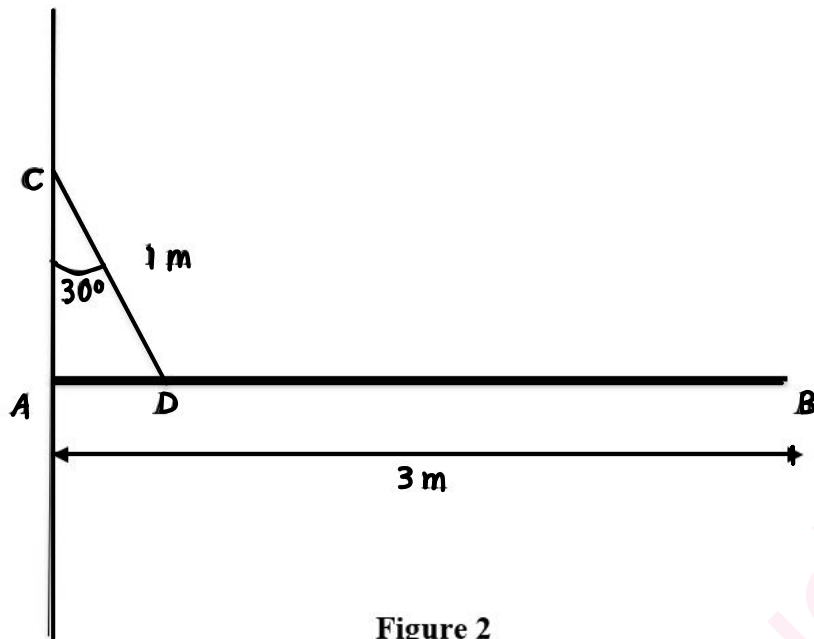


Figure 2

A beam AB , of mass 20 kg and length 3 m, is smoothly hinged to a vertical wall at one end A .

The beam is held in equilibrium in a horizontal position by a rope of length 1 m. One end of the rope is fixed to a point, C , on the wall which is vertically above A . The other end of the rope is fixed to the point D on the beam so that angle ACD is 30° , as shown in Figure 2.

center of mass in the middle

The beam is modelled as a uniform rod and the rope is modelled as a light inextensible string.

Using the model, find

- the tension in the rope, (4)
- the direction of the force exerted by the wall on the beam at A . (6)
- If the rope were not modelled as being light, state how this would affect the tension in the rope, explaining your answer carefully. (2)

The rope is now removed and replaced by a longer rope which is still attached to the wall at C but is now attached to the beam at G , where G is the midpoint of AB . The beam AB remains in equilibrium in a horizontal position.

- Show that the force exerted by the wall on the beam at A now acts horizontally (2)

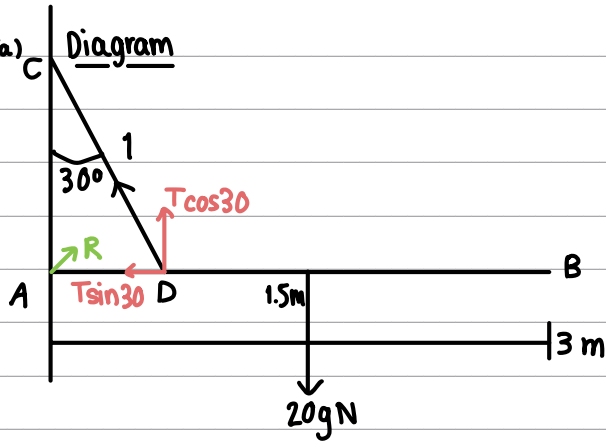
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Question 9 continued

(a) Diagram



★ We take moments about A because this way, the perpendicular distance to R is 0. So we don't need to consider R, making our moments equation much simpler.

Let's use moments to get T:

Taking moments about A: (M1)

$$\sum M_A = 0$$

$$1.5(20g) = AD T \cos 30 \quad (A1)$$

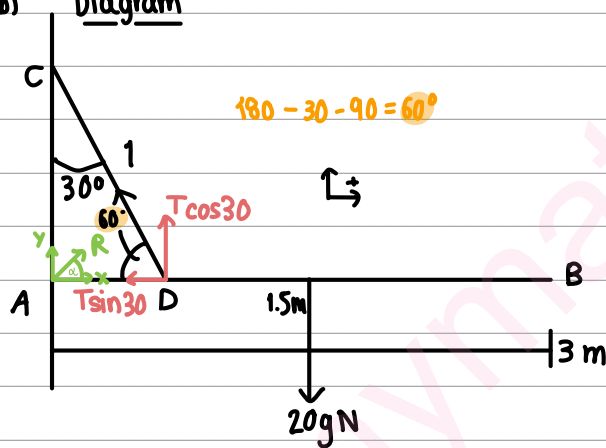
$$AD = 1 \times \sin 30$$

$$30g = \sin 30 \times T \times \cos 30$$

$$30g = \frac{\sqrt{3}}{4} T \quad (A1)$$

$$T = 679 \text{ N to 3sf} \quad (A1)$$

(b) Diagram



We need to get angle α .

Resolve horizontally: (X component of rope) \rightarrow

$$\sum F_x = 0 \quad x = -T \cos 60 = -20g\sqrt{3} \quad (M1A1)$$

Resolve vertically: (Y component of rope) $\uparrow +$

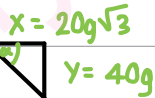
$$\sum F_y = 0 \quad T \cos 30 + Y = 20g \quad (M1A1)$$

$$Y = 20g - T \cos 30 = 20g - 40g\sqrt{3} \cos 30$$

$$= 20g - 60g$$

$$= -40g$$

$\therefore Y$ acts downwards



Now to get α :

$$\tan \alpha = \frac{y}{x} \quad \tan \alpha = \frac{40g}{20g\sqrt{3}} \quad (M1)$$

$$\alpha = \tan^{-1}\left(\frac{40g}{20g\sqrt{3}}\right) = 49^\circ \quad \text{below the horizontal, away from wall} \quad (A1)$$

(c) Tension will increase as you move from D to C, as each point of the rope has to support the length of rope below it. (B1B1)

(d) $\sum M_G$ so we don't have to consider the rope and its tension (M1)

$$1.5 Y = 0 \quad \text{force at wall}$$

$Y = 0 \therefore$ acts horizontally at wall. (A1)

(Total for Question 9 is 14 marks)

10.

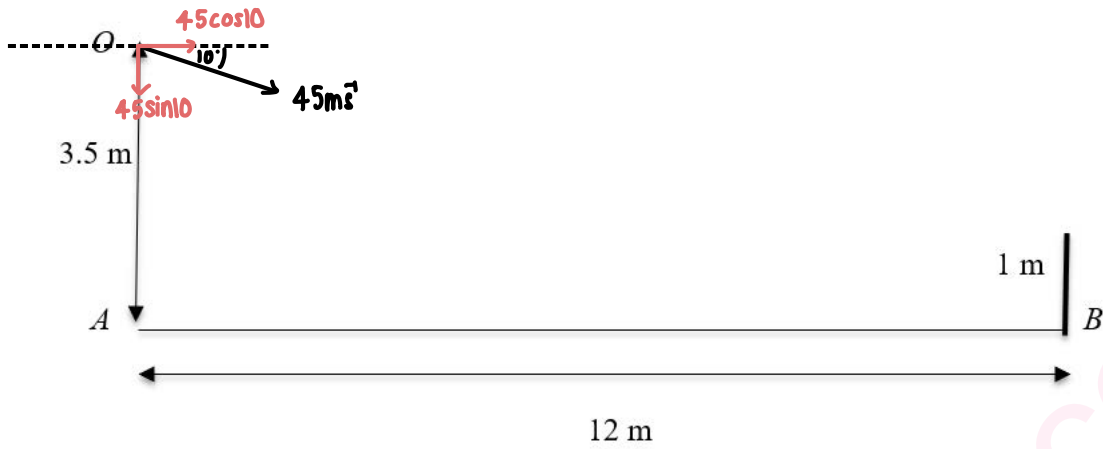


Figure 3

A tennis player serves a ball so as to pass over the net. The ball is given an initial velocity of 45 m s^{-1} in a direction 10° below the horizontal.

The ball is struck at a point O which is 3.5 m vertically above the point A which is on horizontal ground.

The bottom of the net is the point B which is on the ground and $AB = 12 \text{ m}$. The height of the net is 1 m , as shown in Figure 3.

The ball is modelled as a particle moving freely under gravity. The ball passes over the net at a point which is vertically above B .

Using the model,

- (a) find, in centimetres to 2 significant figures, the distance between the ball and the top of the net, as the ball passes over the net, (8)
- (b) find, to 2 significant figures, the speed of the ball as it passes over the net. (4)
- (c) State two limitations of the model that could affect the reliability of your answers. (2)

(a) **Horizontal motion** \rightarrow M1

$s = ut$

$12 = t \times 45 \cos 10 \rightarrow t = 0.2707 \text{ s}$ A1

A1 time at which it crosses the net

Vertical motion: suvat \downarrow - use \downarrow as positive as the motion is \downarrow .

$s = s$

$u = 45 \sin 10$! Hence g is positive

Use formula $s = ut + \frac{1}{2}at^2$ M1

$a = g$ A1 $s = 45 \sin 10 \times t + 4.9t^2$

$t = t$ M1 Substitute t from horizontal motion:

M1 $s = 45(0.2707) \sin 10 + 4.9(0.2707)^2$

convert $m \rightarrow cm$ $100 \times (s - 1) = \text{height above net in cm.}$

2.5 cm to 2sf above net. A1

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Question 10 continued

(b) Horizontal speed is constant: \rightarrow

$$u = 45 \cos 10$$

Vertical motion: $suvat$ $\downarrow +$

$$s =$$

Use formula

$$u = 45 \sin 10$$

$$M1 \quad v = u + at \quad \text{from (a)}$$

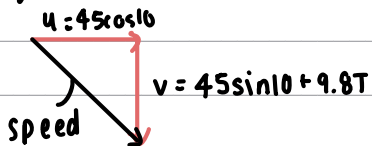
$$v = v$$

$$v = 45 \sin 10 + 9.8t \quad A1$$

$$a = g$$

$$t = t$$

To get the speed use Pythagoras' theorem on the two components



$$|v| = \sqrt{(45 \cos 10)^2 + (45 \sin 10 + 9.8(0.2707))^2} \quad M1$$

$$\text{speed} = 46 \text{ m s}^{-1} \text{ to 2sf} \quad A1$$

(c) The model ignores air resistance $B1$ the model does not consider the size/dimension of the ball $B1$

the model ignores the spin of the ball.

Question 10 continued

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Lined writing area for the answer to Question 10.

(Total for Question 10 is 14 marks)

**TOTAL FOR SECTION B IS 50 MARKS
TOTAL FOR PAPER IS 100 MARKS**